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THE RELATIVE INTENSITIES OF FLUORESCENT AND SCATTERED X-RAYS

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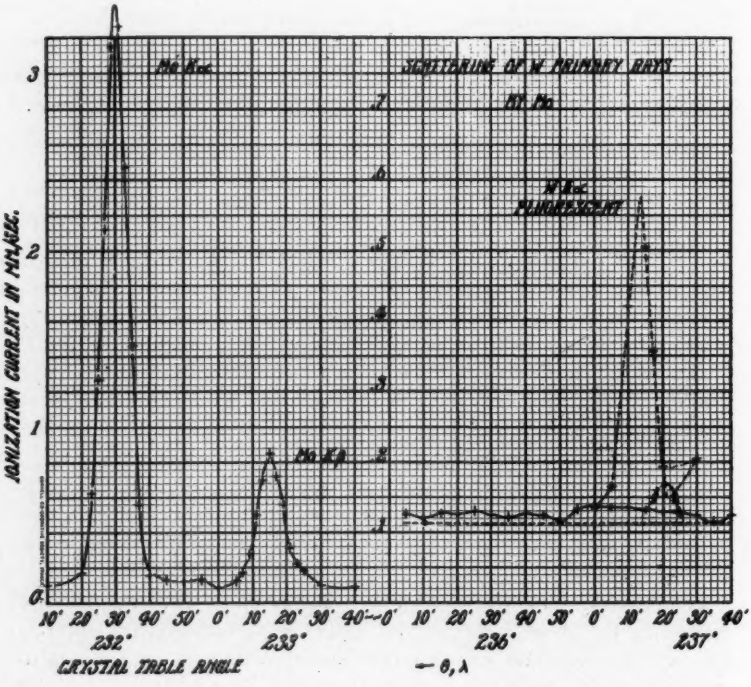
In notes recently published in these Proceedings, we have described experiments on the secondary X-rays from radiators composed of such chemical elements as molybdenum, silver, germanium, etc., due to primary X-rays from a tungsten target. These experiments indicated the presence of scattered tungsten rays and tertiary radiation of almost the same order of intensity as the fluorescent radiation characteristic of the chemical elements composing the secondary radiators. Experiments were reported by Allison and Duane in the January, 1925, number of these Proceedings in which they observed the scattered radiation, including the Compton effect, but only very weak radiation with no sharply marked peaks at the points in the spectra where the tertiary radiation should lie. The experiments described in this note have been performed for the purpose of verifying previous results, particularly with regard to the question as to whether the scattered and tertiary radiation from chemical elements of medium and high atomic weights have really the same order of intensity as the fluorescent radiation.

Figure 1, page 414 of the December, 1923, number of these Proceedings represents the arrangement of the apparatus. Increased lead protection has been provided for the quadrant electrometer and for the ionization chamber where the battery wires enter it. This additional protection reduces the magnitudes of the ionization currents to a considerable extent, presumably due to the cutting off of stray radiation and of radiation scattered in all directions from the crystal.

The full line curve in the figure represents the results of an experiment with a molybdenum secondary radiator and primary radiation from a tungsten target. The peaks marked Mo$K\alpha$ and Mo$K\beta$, corresponding to the fluorescent radiation of molybdenum, are much taller than in the previous experiment and the $K\alpha_1$ and $K\alpha_2$ lines are not separated from each

other. This is due to the fact that broader slits were used than in the previous experiments. The scattered radiation, not only near the molybdenum fluorescent peaks, but also at the tungsten K series lines, and the tertiary radiation are very weak. In fact the intensity is so small that we cannot measure it with any degree of certainty. The horizontal straight line in the figure represents the ionization current obtained when the crystal has been placed in such a position as not to reflect the rays into the ionization chamber. The height of this line, therefore, represents the influence of stray radiation, etc. The observed currents produced when



the crystal is in position to reflect the rays into the ionization chamber are only a very little greater than those represented by this straight line. Hence the scattered and tertiary radiation are of a different order of intensity from the fluorescent.

The curve marked W$K\alpha$ represents the *fluorescent* $K\alpha$ doublet from a *tungsten* secondary radiator. It serves to mark the position of the tungsten $K\alpha$ lines in the spectrum. Its intensity is very much greater than that due to the $K\alpha$ tungsten radiation scattered from *molybdenum*, although

experimental conditions such as current, voltage, slit widths, etc., remain the same.

Results similar to the above have been obtained for secondary radiators composed of other chemical elements having atomic weights near that of molybdenum.

When chemical elements of low atomic weight such as carbon were used as secondary radiators we obtained ionization currents of about 0.1 or 0.2 millimeter per second representing the scattered tungsten $K\alpha$ radiation. In other words, the scattered radiation from the light chemical elements has a very much greater intensity than from the heavy, which agrees with previous findings.

The conclusions are that the scattered and tertiary radiation due to tungsten X-rays falling upon chemical elements of atomic weights near that of molybdenum are extremely weak as compared with the fluorescent radiation. They are too weak to be accurately studied with our present apparatus. The large currents in former experiments representing the scattered and tertiary radiation must have been due to some other cause than the radiation coming from the secondary radiator. The general stray radiation will not explain the tungsten peaks observed in the previous experiments and we do not know where they came from.

These very weak spectra will be investigated more thoroughly as soon as the new high tension storage battery has been completed and as soon as a water-cooled tungsten target tube which the General Electric Company kindly furnished us and which was accidentally burnt out has been repaired.

NOTE ON THE QUANTUM THEORY OF THE REFLECTION OF X-RAYS

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In a note published in these PROCEEDINGS for May, 1923, the writer proposed the fundamental principle of a theory of interference and diffraction phenomena based on quantum laws. According to this theory, when a quantum of radiation changes its direction in accordance with the observed phenomena of interference and diffraction, it transfers energy and momentum to the diffracting system. The transfer of momentum, according to the theory, takes place in quanta, the magnitudes of the quanta depending in a particular way upon the characteristics of the diffracting system.

This theory has been developed further by Breit,[1] A. H. Compton[2] and Epstein and Ehrenfest,[3] the latter having extended it to Fraunhofer diffraction in general.

In the original article, the theory was applied firstly to the reflection of an X-ray by a crystal in which the X-ray does not give up energy to an atom in the reflecting crystal, splitting off an electron. Then the case in which the X-ray does split off an electron and the atom subsequently emits some of the lines in its own spectra was discussed briefly and equations were deduced which indicate a rather strong reflection by the crystal of its own characteristic radiation. The object of this note is to examine the latter application of the theory in greater detail.

In order to apply the theory to an incident X-ray which splits off an electron and causes an atom in the crystal to radiate its fluorescent spectrum, an assumption had to be made, in addition to the fundamental laws of the theory. This additional assumption was that the total fluorescent radiation from an atom quantized its momentum with the crystal, the size of the quantum being determined, as in the general case, by the characteristics of the crystals. With this additional assumption the theory leads to the equations (7) of the original note. In general, they represent three-dimensional problems, but, for simplicity, we may consider an approximate, two-dimensional one. For reflection in a particular case the equations (7) reduce to

$$\begin{aligned} \Sigma \frac{1}{\lambda_1} (\cos\theta - \cos\theta_1) &= \frac{\tau_1}{d_1} \\ \Sigma \frac{1}{\lambda_1} (\sin\theta - \sin\theta_1) &= \frac{\tau_2}{d_2} \end{aligned} \qquad (1)$$

where θ is the glancing angle of incidence; θ_1, θ_2, etc., the angles of reflection of the various fluorescent rays, having the parameters that we call wave-lengths λ_1, λ_2, etc. d_1 and d_2 are two of the grating spaces in the crystal, and τ_1 and τ_2 are whole numbers, whose values determine the particular sets of planes in the crystal that do the reflecting and the orders of the reflections from them. Taking λ_1 to refer to the fluorescent ray of greatest energy emitted by the atom it appears that in any particular case, with τ_1 and τ_2 given, θ_1 can have only certain values whereas θ_2, θ_3, etc., are indeterminate. Assuming that all values of θ_2, θ_3, etc., are equally probable, that is, that the corresponding X-rays have the same probability of passing off from the crystal in all directions, we can calculate the possible values of θ_1 and also estimate the probability that θ_1 will have any one of its possible values.

Figure 1 represents graphically the relation between the angle of incidence θ and the possible values of θ_1 corresponding to it for the particular case $\tau_1 = 0$, that is, for reflections from let us say the 100 planes of the

crystal. It will be seen that the points representing corresponding values of θ and θ_1 lie on a closed ellipse-shaped curve and that they are more or less concentrated about the ends of the curve. One end, A, of the curve lies

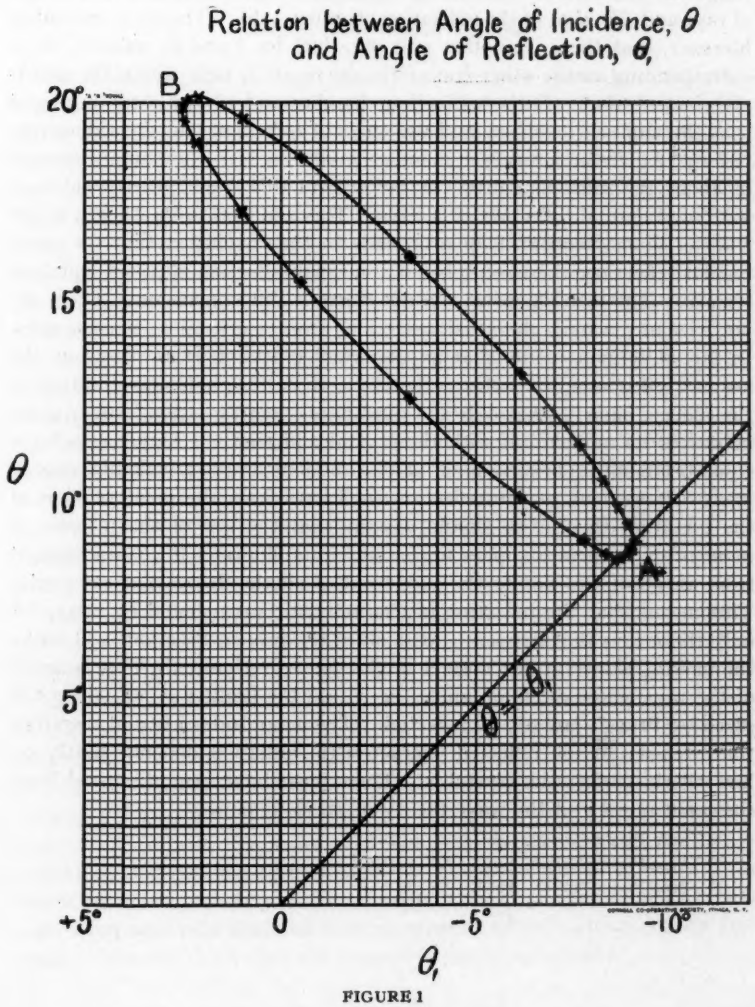

FIGURE 1

very close to the straight line representing $\theta = -\theta_1$, which corresponds to normal reflection of X-rays from the planes of the crystal under consideration. If an ionization spectrometer were set up with the angle of inci-

dence, θ, equal to the angle of reflection, θ_1, it would register a maximum ionization current when this common angle had the proper value; for the current would be proportional to the number of points in the small rectangle $\Delta\theta \times \Delta\theta_1$, $\Delta\theta$ representing the angular width of the incident beam of rays and $\Delta\theta_1$, that of the ionization chamber's slit. The curve indicates, however, that there is another pair of values for θ and θ_1, namely, those corresponding to the other end of the curve at B, near which the points are concentrated. If the ionization chamber and crystal were so placed that the angle of incidence and the angle of reflection had the values represented by B the ionization chamber would register a maximum current of about the same magnitude as before and this would correspond to a kind of abnormal reflection of X-rays. This addition to the theory of the transfer of momentum in quanta leads to the conclusion that for every regular reflection, there must be an abnormal reflection of about equal intensity. The facts, however, do not warrant this conclusion. There are not as many strong abnormal reflections from a crystal as normal ones. In fact it seldom occurs that an abnormal reflection is as strong as the normal reflections. It follows, therefore, that this particular addition to the theory, namely, that only the total fluorescent radiation from an atom quantizes its momentum with the crystal and that the longer rays have equal probabilities of passing off in all directions, is not justified by experiment. If we look at the reflection of X-rays from the point of view of the quantum theory, we should assume that the law of the transfer of momentum in quanta applies to the individual fluorescent quanta emitted from an atom and not to the sum total of all the fluorescent rays only. This means that the reflection of characteristic rays should be weak, for only those rays that happen to pass off in the proper directions will strike the crystal planes at the correct angles for reflections. A large number of the rays whose direction angles lie within the limits $\theta \pm \delta\theta$, where θ is given by Bragg's law will be reflected. The magnitude of $\delta\theta$ and therefore the intensity of the reflected characteristic radiation depends partly on how nearly perfect the crystal is. These ideas have been discussed from the point of view of the wave-theory in previous papers.

[1] These PROCEEDINGS, July, 1923, p. 238.

[2] These PROCEEDINGS, Nov., 1923, p. 359.

[3] These PROCEEDINGS, April, 1924, p. 133.

A NEW PRINCIPLE OF EQUILIBRIUM

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The Non-Existence of Cyclic Equilibrium.—At a given temperature a system is said to be in a state of physical and chemical equilibrium if none of the describable properties of the system change, on the average, with the time. Any given substance may be regarded as made up of *groups*, each group being defined by certain macroscopic or microscopic properties. Thus a heterogeneous substance such as a mixture of ice and water naturally divides itself into the two groups ice and water, but each of these may be divided into sub-groups. Thus the liquid water may contain several molecular species, such as H_2O, $(H_2O)_3$, H^+, H_3O^+, OH^- and each of these species may be taken as a group. The molecules of each such species may be further divided. One group may consist of all the molecules of the species H_2O located within a certain cubic centimeter, and having such motion of translation that the component in an X direction lies between certain specified limits, that in the Y direction between other specified limits, and so on. (By taking these limits as close together as we please, we may diminish the *average* content of any one group without limit.) Every group so defined would be still further sub-divided if we should specify the positions and the motions of the several particles of which the molecule itself is composed.

When once we have specified the characteristics of a group, no matter how closely it may be delimited, the average number of molecules which it contains must remain constant when equilibrium is attained. In other words, the average number of molecules which in unit time cease to belong to a given group must be equal to the average number of new molecules entering that group.

In some cases thermodynamics gives more information than this concerning equilibrium. Let us consider a system of ice, water and water vapor, and indicate by s_{wi}, s_{iw}, s_{vw}, s_{wv}, s_{vi}, s_{iv} the average number of molecules changing per second from water to ice, from ice to water, from vapor to water, and so on. The definition of equilibrium which we have stated requires only that the average amount of each group remain constant. The condition that the amount of ice be constant is

$$s_{iv} + s_{iw} = s_{vi} + s_{wi}$$

It does not prohibit a condition of cyclic equilibrium such that

$$s_{iw} > s_{wi};\ s_{wv} > s_{vw};\ s_{vi} > s_{iv}$$

Nevertheless we know that such a condition is impossible, for the ice may be removed from the system in such a manner as neither to change the relative rates of vaporization of the liquid and of liquefaction of the vapor, nor to disturb the equilibrium between these two groups. But after the ice is removed we must have s_{iw} equal to s_{vw}, and therefore this must have been the condition before the removal of the ice.

Let us suppose that an equilibrium exists between two chemical substances $A \rightleftharpoons B$, and that there are two paths by which A can be converted into B and two corresponding reverse paths. The four rates of reaction may be expressed by s_{AB}, s_{BA}, s'_{AB} and s'_{BA}. Often we know that we cannot have a cyclic equilibrium such that

$$s_{AB} > s_{BA};\ s'_{AB} < s'_{BA},$$

but rather must write

$$s_{AB} = s_{BA};\ s'_{AB} = s'_{BA}.$$

For if this were not the case we could add a minute amount of some catalyst which would increase the rate of the reaction and its inverse along one of the paths, without affecting the two rates in the other path. This would disturb the existing equilibrium, contrary to the results of observation and of thermodynamics.

In the absence of catalytic phenomena we are still led to similar conclusions. Let us consider Wegscheider's paradox.[1] He writes, for example, the simultaneous reactions,

$$A \rightleftharpoons B;\ B + C \rightleftharpoons A + D;\ D \rightleftharpoons C,$$

and points out that if we impose the sole condition that the amount of each substance is constant, the ordinary laws of chemical kinetics lead to a different equation for the equilibrium concentrations than the one obtained from thermo-dynamics; it reduces to the latter only when certain further conditions are imposed. The assumption that the rate in each individual reaction is equal to the inverse rate removes the paradox. This is not the only way of avoiding the paradox in this particular example, but if we were to consider a great many different types of simultaneous reactions we should probably find that the only general way of avoiding the paradox, that is applicable to all cases, would be to assume that in every case of equilibrium, involving a number of simultaneous reactions, a state of equilibrium also exists for each separate reaction and its inverse.

In the more purely physical types of equilibrium, such as are dealt with in statistical mechanics, we know in certain simple cases not only that the number of molecules in a certain limited group remains constant, but that, for example, the number of molecules in a certain group L colliding with those in a group M to give molecules in groups L' and M' is equal to the

number of molecules in groups L' and M' colliding to give molecules in groups L and M.[2] Here again we are tempted to generalize and assume that the same principle holds for all cases.

Finally let us turn to the phenomenon of absorption and emission of light. If it be assumed that the emission of a certain spectral line indicates that some molecules have undergone a process by which they have passed from one group to another, and if we assume that when the same line is absorbed the molecules undergo the reverse process and are transferred from the second group to the first, it seems inevitable that in a state of thermal equilibrium the two processes must occur at the same average rate. Otherwise it would be hard to see how such a substance could be introduced into an isothermal Hohlraum without permanently changing the character of the radiation therein.

These diverse examples suggest very strongly that cyclic equilibrium does not exist, and we have seen that this supposition could be proved correct if an independent catalyst could be found for each individual reaction or process. There are, however, numerous cases in which it would be difficult, if not impossible, to find such catalysts. In applying thermodynamics to solutions, some of the more important results have been obtained by the use of a semi-permeable membrane. Now there are solutions in which it would be difficult, if not impossible, to find such a membrane, but we do not hesitate to apply to them the same laws which can be demonstrated to be true for cases where semi-permeable membranes can be employed. So in our present case we should hesitate to believe that there are different laws for reactions which are subject to catalysis and for those which are not.

The Law of Entire Equilibrium.—Thus I am led to propose a law which in its general form is not deducible from thermodynamics, but proves to be compatible with the laws of thermodynamics in all cases where a comparison is possible. It may be called the law of *entire equilibrium*, and may be stated as follows. *Corresponding to every individual process there is a reverse process, and in a state of equilibrium the average rate of every process is equal to the average rate of its reverse process.* The rate at which one group or set of groups L goes over into another group or set of groups L' is the same as the rate at which the groups L' go over into the groups L. Moreover if there are various paths by which the first process occurs, there is an equal number of paths by which the second process occurs, and the rate is the same in both directions along every path. This will be true no matter how detailed are the specifications which define the several groups and the several paths.

In cases to which pure quantum theory may be applied there is a number of discrete paths by which molecules of one group may change to another. Thus in Bohr's theory of the hydrogen atom, the atom may pass from a

higher to a lower quantum state, either immediately or through a finite number of intervening states. The meaning of the principle is here obvious; in a condition of thermal equilibrium the rate of change for each step is equal to the rate for the reverse step.

In other cases we must assume, not a discrete number, but an infinite continuum of paths. Then we must employ the term "path" to mean some specified zone of paths more or less arbitrarily demarked from the continuum. Yet however narrow or however arbitrary this path, or zone of paths, is, our principle asserts that the rate along this path is the same in both directions. For example, in discussing statistical mechanics it is often convenient to plot the position of each particle in a "phase space" of as many dimensions as there are numbers required to specify the position and the momentum of one particle. As a particle changes its position and momentum, it describes a curve in this space, and we may segregate for discussion all of these curves which lie within a certain "tube." Let us suppose that two molecules collide in such manner that the molecule A goes from the group L to the group L', and the molecule B from the group M to the group M', and further that each molecule traces a curve lying within a specified tube, one from L to L' and one from M to M'. Then our law states that in equilibrium the average rate (or chance) of a molecule A going, within the prescribed tube, from L to L' while a molecule B goes through its prescribed tube from M to M' is the same as the average rate (or chance) of molecules A and B going through the same tubes from L' to L and from M' to M.

Perhaps the significance of this law may be made a little more evident by means of a crude analogy. Suppose that during a period in which there are no births or deaths the population of the several cities of the United States remains constant, the number leaving each city being balanced by the number entering it. This stationary condition would not correspond to our case of thermal equilibrium. We should require further, to complete the analogy, that as many people go from New York to Philadelphia as from Philadelphia to New York. If there were three railway lines between these two cities we should require that the number of passengers going by each line be equal in both directions. If some of the travel were not by railway or roads but across country, then, if we should draw on the map two non-intersecting lines from New York to Philadelphia, we should require that the number of persons passing through any such zone in one direction be equal to the number passing through the same zone in the opposite direction. By such illustrations we may appreciate how extremely far-reaching are the consequences of the proposed law.

The law of entire equilibrium might have been called the law of reversibility to the last detail. If we should consider any one of the elementary processes which are occurring in a system at equilibrium, and could, let us say, obtain a moving-picture film for such a process, then this

film reeled backward would present an equally accurate picture of a reverse process which is also occurring in the system and with equal frequency. Therefore in any system at equilibrium, time must lose the unidirectional character which plays so important a part in the development of the time concept. In a state of equilibrium there is no essential difference between backward and forward direction in time, or, in other words, there is complete symmetry with respect to past and future.

I believe that some of the ideas contained in this paper have been suggested by the work of Einstein, but he has not proposed this law of equilibrium. Indeed one of the first applications which I shall make, in a subsequent paper, will be to the interaction between matter and light, where I shall attempt to demonstrate the invalidity of Einstein's derivation of Planck's radiation formula. Another application will shortly appear in a paper by Dr. D. F. Smith and myself on the mechanism of chemical reactions.

[1] Wegscheider, *Z. physik. Chem.*, **39**, 273 (1901).

[2] See Lewis and Adams, *Physic. Rev.* **5**, 10 (1915).

STUDIES ON DISEASE RESISTANCE IN THE ONION

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The nature of resistance to disease in plants is a subject which has been of increasing interest to botanists in general and to plant pathologists in particular for several years past.[1] Earlier there was a tendency to assume that in various instances of disease resistance the causes were sufficiently similar to justify some single type or comparatively few types of explanation, as appears to be the case in certain cases of immunity in animals. Investigations indicate, however, that the causes of resistance in plants are widely varied and relatively complex, and that sound progress is dependent, for the present at least, upon critical study of specific examples wherever practicable. Some years ago it came to the writer's attention that the bulbs of the common onion show a wide range in susceptibility or resistance to certain fungous parasites and that this apparently is correlated with the occurrence of scale pigments. The studies summarized here are therefore presented as a contribution to the general subject of disease resistance in plants.

Horticultural varieties of the common onion (*Allium cepa*) are grouped

into three classes with respect to the occurrence of pigment dissolved in the cell sap of the outer epidermal layer of the bulb. In one class, the white varieties, such pigment is absent or practically so. In another class, the yellow varieties, there is a light yellowish tinge to the fleshy scales and with the proper reagents the presence of flavone derivatives is readily demonstrated in the outer epidermis. The dry outer scales are of a deeper yellowish brown color, the intensity of which differs with the variety. In the third class, the red variety, the epidermal cells give positive micro-chemical tests for anthocyan and these tests indicate a certain amount of flavone derivatives also present. The dry outer scales are of a purplish red color.

The common names and causal fungi of the diseases considered are as follows: smudge (*Colletotrichum circinans* (Berk.) Vogl.),[2] neck rot (*Botrytis allii* Munn. and *Botrytis sp.*) and black mold (*Aspergillus niger* van Tieg.). All of these organisms subsist for a part of their life cycle saprophytically on dead organic material in or upon the soil. While they do not ordinarily invade the growing onion plant, they do often attack the dead outer scales before harvest. As soon as the plant is mature they infect the living fleshy storage tissues of the scales, causing gradual decay during storage. The smudge organism attacks the bulbs at any point on the outer scale, lives for a time saprophytically on this dead tissue, and as it reaches the fleshy scale penetrates the cuticle directly. The black-mold organism attacks in much the same manner. Both of these organisms penetrate the fleshy tissue very gradually as compared with the neck-rot fungi. The latter, though capable of living on the dead outer scales, do not ordinarily occur there except on the tissue at or above the neck of the bulb. As the tops die down, the neck-rot fungi invade those tissues and thereby gain entrance to the succulent scales whence they proceed downward rather rapidly. Considerable moisture is necessary to facilitate infection by these Botrytis species and as a rule a general epiphytotic of neck rot is experienced only when abundant rainy weather prevails at the harvest period.

Resistance of colored varieties of onion—both yellow and red—to smudge becomes evident some weeks before harvest. When they are grown on infested soil alongside white varieties, the disease becomes evident as black smudgy spots over the entire outer scales of the latter, even extending, under optimum conditions of moisture, to the sheath about the neck. Little or no infection is found on either red or yellow bulbs exposed to similar environment—except under optimum conditions when spots are found on the uncolored outer leaf tissue above the neck. This is strictly limited, however, to the region above the color. This tissue is largely removed at harvest and thus even under weather conditions extremely favorable to the disease the colored bulbs remain remarkably free from smudge through-

out the remainder of the growing season and through storage. The disease advances steadily in the white variety from the time of initial infection.

The black mold disease is considered here by way of contrast. The fungus, attacking the plants a little later but in somewhat the same manner as smudge, is quite impartial to varieties as far as color is concerned. No consistent evidence of varietal resistance is to be noted.

In case of neck rot the white varieties are again always the more susceptible. The mode of attack is distinct from the other two organisms, in this case the succulent neck tissue being the one in question. Under moderately favorable conditions for infection, white varieties are often severely affected, while colored varieties remain quite free from the disease. Under the most favorable conditions for infection, however, the disease is often of consequence in the colored bulbs, but the degree of difference between them and the white bulbs remains essentially the same.

We thus have a distinct correlation between the flavone or anthocyan color in onion bulbs and their resistance to two common diseases, smudge and neck rot, while with a third disease under consideration, black mold, this difference from uncolored bulbs does not hold.

In an attempt to determine the inherent difference between colored and white varieties which contributes toward resistance in the former, attention was first given to the growth of the fungi in the expressed juice of succulent onion scales. It was found that although the undiluted extract retarded growth of all the organisms, no outstanding difference in this respect was to be found between that of colored and uncolored bulbs. The qualities of the succulent scales were therefore not to be expected to account for the greater resistance of colored bulbs. This conclusion was strengthened by the results of another experiment. The dry outer scales of yellow and red varieties were removed and inoculum containing spores of *Colletotrichum circinans* were placed directly on the succulent scales. Under such conditions, even the colored bulbs were readily infected. Similarly, when spores and mycelium of Botrytis were injected into wounds made in the necks of colored and white bulbs, they all decayed with about equal rapidity. Thus, when the infecting hyphae of either the smudge or the neck rot fungi once gain entrance to the succulent scales of colored bulbs, the resistance of the host no longer obtains.

It was therefore assumed that the resistant quality was to be found active in the dead outer scales which the parasite normally enters first before invading the living cells. The cells of the outer scales being dead, any water-soluble material present in the tissue would be expected to diffuse readily into drops of water which come into contact with them. Thus, spores or mycelium of either of these fungi, as they come in contact with the host, are exposed to any material which dissolves out of the outer scales. The study was therefore directed to a consideration of the effect

of cold water extracts from dry outer scales of white, yellow and red bulbs upon the germination and growth of the fungi concerned.

Water extracts were made from the dry outer scales of red, yellow and white bulbs by adding approximately equivalent amounts of each to drops of water containing spores of the fungus concerned. The first and most extensive studies were made with the smudge organism. Normal germination and growth took place when white scale tissue was added to the germination drops. In drops to which either red or yellow tissue had been added, a decided toxic effect on the organism was noted. The abnormal response of the fungus became manifest in several forms. In some cases the spores were killed without any evidence of germination. Very often the spore enlarged perceptibly, often becoming septate, but producing

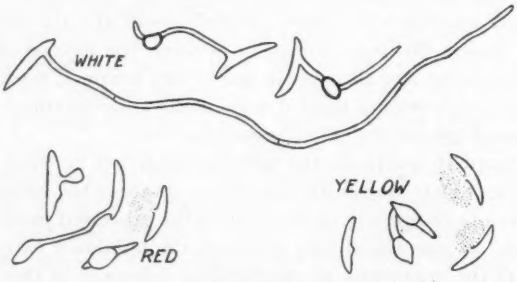

FIGURE 1

Effect of water extract of outer onion scales upon spore germination of the smudge fungus. Bits of dry scales of white, yellow and red onions were added to drops of spore suspension in distilled water. Note typical germ tubes and appressoria in the case of the white scale as compared with abnormal germination in the case of the colored scales. The abnormalities are of three types: (1) ruptured germ tubes, (2) short thick germ tubes, (3) swelling of spores without production of germ tubes.

no germ tube. Occasionally short, distinctly retarded germ tubes were formed. In the majority of cases the beginnings of germination were made but as soon as the germ tube had started to form, the wall at the growing tip disappeared while the cytoplasmic contents of the spore exuded through this opening into the liquid, thus precluding further functioning of the parasite. The results of a typical experiment showing the percentage of spores reacting in the various ways are given in table 1, and camera lucida sketches of spores after exposure to the various extracts are reproduced in figure 1. The presence in outer colored scales of a substance decidedly toxic to the smudge organism is clearly demonstrated, while in the white varieties this toxin is absent, or present in such small quantities as to be

negligible. This substance dissolves readily in water and therefore functions when the organism comes into contact with colored scales, and when there is sufficient moisture present which, of course, would be necessary to infection.[3]

TABLE 1

EFFECT OF DRY OUTER SCALE TISSUE OF RED, YELLOW AND WHITE VARIETIES OF ONION UPON THE GERMINATION OF COLLETOTRICHUM CIRCINANS

COLOR OF TISSUE USED	SPORES GERMINATING NORMALLY (PER CENT)	SPORES NOT GERMINATING (PER CENT)	SWOLLEN SPORES (PER CENT)	SHORT ABNORMAL GERM TUBES (PER CENT)	SPORES EXUDING CATOPLASM (PER CENT)
Control	91	9	0	0	0
Red	2	7	4	42	45
Red	0	12	0	0	88
Yellow	0	34	0	0	66
Yellow	0	24	0	0	76
White	92	8	0	0	0
White	87	13	0	0	0

The same type of experiment was conducted with the two neck-rot fungi and with the black-mold organism. The smudge organism was included for comparison. The data given in table 2 show the percentage of spores producing apparently normal germ tubes and the average length of those germ tubes after 24 hours. It will be seen that the results with *C. circinans* confirm those previously given in table 1. In the case of the two neck-rot organisms, germination and growth in white scale extract was equal to or better than the control. In the colored scale extract, with the exception of *Botrytis sp.* in yellow extract, the germination was very poor and growth very slight. In the exception just noted, although 47% of the spores germinated, the germ tubes grew very little. In contrast to the smudge and neck-rot organisms the germination of the black-rot organisms was greatly increased over the control in both colored and white scale extract and the rate of growth was infinitely better than the control in all three extracts.[4]

From these results it is concluded that the resistance of colored bulbs to smudge and neck rot is due to a readily soluble substance in the dry outer scales of colored varieties which is toxic to the causal organisms of the diseases concerned. The substance becomes functional by dissolving into the infection drop and inactivating the fungus before penetration can take place. The fact that this substance does not affect *Aspergillus niger* would seem to account in part at least for the lack of resistance in colored varieties to black mold.

The nature of this inhibitive substance constitutes a problem of exceeding interest. Its very close association with the scale pigment has already been shown while no evidence of its existence in uncolored tissue has been found. This is readily shown by adding to one germination drop portions

of an outer scale taken from above the neck where the color does not develop and to another drop tissue of the same scale taken from just below the point where pigment is present. In the first, normal germination occurs; in the second, the typical toxic effects upon the organism result. It may be supposed that the coloring substance is actually the toxic substance. Although this is possible the final proof of the case is not easily attained.

TABLE 2

EFFECT OF DRY OUTER SCALE TISSUE OF RED, YELLOW AND WHITE VARIETIES OF ONION UPON THE GERMINATION AND GROWTH OF THE CAUSAL ORGANISMS OF SMUDGE (COLLETOTRICHUM CIRCINANS), NECK ROT (BOTRYTIS ALLII AND BOTRYTIS SP.), AND BLACK MOLD (ASPERGILLUS NIGER)

ORGANISMS	GERMINATION (PER CENT)				AVERAGE LENGTH OF GERM TUBES (MICRONS)			
	CONTROL	WHITE	YELLOW	RED	CONTROL	WHITE	YELLOW	RED
Colletotrichum circinans	99	80	0	2	414	224	0	1—
Botrytis allii	70	68	1	1	491	554	5	28
Botrytis sp.	72	99	47	3	24	146	8	1—
Aspergillus niger	29	93	59	53	6	129	95	118

The fact that there is no constant difference between the amount of toxic material in red and yellow scales, together with the fact that flavones exist in both types of scale, would indicate the flavone derivatives rather than the anthocyan to be more probably the one concerned. The basic flavone derivative in colored onion scales is quercetin. This is found in the free state and combined with glucose in both the yellow and red scales. The nature of the anthocyan in the red scales is not known. Free quercetin, when extracted and purified, is only very slightly soluble in cold water and it has not been possible to duplicate the toxic effect upon the fungi with any solution of quercetin obtainable. Likewise, any glucoside of quercetin which has been isolated and chemically purified is very difficultly soluble in water.

It is possible that the toxic substance is a soluble derivative of quercetin, probably a glucoside, which is not readily isolated in its original form by ordinary chemical methods and which thus becomes less soluble in water due to loss of part of the glucose radicals. If this takes place in chemical extraction, the exact duplication of the results secured with crude water extract of colored scales becomes the more difficult.

[1] For accounts of other work on disease resistance in plants at the University of Wisconsin, see Jones, L. R.: "Disease Resistance in Cabbage," *Proc. Nat. Acad. Sci.*, **4**, 42–6(1918); Dickson, J. G., et al.: "The Nature of Resistance to Seedling Blight of Cereals," *Ibid.*, **9**, 434–9 (1923). For a general review of the literature on the subject see Walker, J. C.: "On the Nature of Disease Resistance in Plants," *Trans. Wis. Acad. Sci. Arts and Letters*, **21**, 225–247 (1924).

[2] For a detailed description of smudge see Walker, J. C.: "Onion Smudge," *J. Agr. Res.*, **20**, 685–722 (1921).

[3] For a more detailed account of studies of resistance to smudge see Walker, J. C., "Disease Resistance to Onion Smudge," *J. Agr. Res.*, **24**, 1019–40 (1923).

[4] See further, Walker, J. C., and Lindegren, C. C.: "Further Studies on the Relation of Onion Scale Pigmentation to Disease Resistance," *J. Agr. Res.*, **39**, 507–514(1924).

NOTES ON STELLAR STATISTICS. IV. ON THE RELATION BETWEEN THE MEAN VALUES OF THE v AND τ COMPONENTS OF PROPER MOTION.

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In a previous paper[1] we have discussed the frequency function of velocities at right angles to the line of sight, under the assumption that the distribution of cosmic velocities can be represented by a single Maxwellian function with a drift superposed. It may sometimes be useful to consider also the distribution of the separate components of this tangential velocity and their arithmetic mean values.

Let the tangential velocity be resolved into two components, the v component directed toward the apex of the drift, and the τ component at right angles to this direction. Consider a small area in the sky, situated at an angular distance λ from the apex of the drift, and denote by V the speed of the drift and by h the constant of the Maxwell distribution. The frequency law of the τ components remains unchanged and the arithmetic mean value τ_a is equal to $\tau_a = 1/h\sqrt{\pi}$. The distribution of the v velocities is given by:

$$f_\lambda(v)dv = \frac{h}{\sqrt{\pi}} \exp\,[-h^2(v - V\sin\lambda)^2]dv$$

and the arithmetic mean value v_λ may be calculated from:

$$v_\lambda = V\sin\lambda\,\mathrm{erf}(hV\sin\lambda) + \frac{1}{h\sqrt{\pi}}\exp(-h^2V^2\sin^2\lambda)$$

The mean value v_a, of v throughout, i.e., taken over the whole sphere, is subsequently determined by:

$$v_a = \int_0^{\pi/2} v_\lambda \sin\lambda d\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

When dealing with giant and slow-moving stars, hV is probably less than 1.7, and accordingly H. C. Plummer's[2] approximation to the error function

$$\mathrm{erf}\,(x) = \frac{6}{\sqrt{\pi}}\frac{x}{3 + x^2}$$

may be used to obtain a formal solution of (1)

$$v_a = \int_0^{\pi/2} \left[V \sin\lambda \cdot \frac{6}{\sqrt{\pi}} \cdot \frac{hV \sin\lambda}{3 + h^2V^2 \sin^2\lambda} + \frac{1}{h\sqrt{\pi}} \exp(-h^2V^2 \sin^2\lambda) \right] \sin\lambda d\lambda$$

Putting $u = hV$, $a = \sqrt{1 + 3/u^2}$, remembering that $\tau_a = 1/h\sqrt{\pi}$, and performing the integration, we obtain

$$v_a/\tau_a = \frac{e^{-u^2}}{u} \int_0^u e^{x^2}\, dx + 6 \left(1 - \frac{3}{2au^2} \log \text{nat} \frac{a+1}{a-1} \right) \ldots\ldots (2)$$

In a similar way we can derive the distribution function and the arithmetic mean value, R_a, of the uncorrected radial velocity, in terms of the arithmetic mean corrected radial velocity v_a. When expressed in the same unit we have of course $v_a = \tau_a$; we have here employed different symbols because τ is generally expressed in seconds of arc and v in kilometers per second.

We thus have

$$R_\lambda = V \cos\lambda \operatorname{erf}(hV \cos\lambda) + \frac{1}{h\sqrt{\pi}} \exp(-h^2V^2 \cos^2\lambda)$$

Writing $q = hV/\sqrt{3} = u/\sqrt{3}$, we finally obtain for R_a/v_a the expression

$$R_a/v_a = \frac{1}{u} \int_0^u e^{-x^2}\, dx + 6 \left(1 - \frac{1}{q} \operatorname{arc\,tan} q \right) \ldots\ldots (3)$$

Equations (2) and (3) give the formal solutions of our problems with sufficient accuracy in the case that $u < 1.7$. For purposes of computation and interpolation, however, it appears preferable to expand the integrands in power series for all integrations. Choosing as the independent variable, in which to express the result, the ratio w between the drift (solar) velocity and the arithmetic mean, θ, of the Maxwell distribution (in practice corresponding to the arithmetic mean corrected radial velocity, $\theta = V_a = \tau_a$) we are able to write (2) and (3) in the forms:

$$w = V/\theta = hV\sqrt{\pi}$$

$$v_a/\tau_a = 1 + \frac{1}{1.3.1} \frac{2w^2}{\pi} - \frac{1}{1.3.5.3} \frac{4w^4}{\pi^2} + \frac{1}{1.3.5.7.5} \frac{8w^6}{\pi^3} - \frac{1}{1.3.5.7.9.7} \frac{16w^8}{\pi^4} \ldots (4)$$

$$R_a/v_a = 1 + \frac{1}{1.3.1!} \frac{w^2}{\pi} - \frac{1}{3.5.2!} \frac{w^4}{\pi^2} + \frac{1}{5.7.3!} \frac{w^6}{\pi^3} - \frac{1}{7.9.4!} \frac{w^8}{\pi^4} \ldots (5)$$

or, numerically:

$$v_a/\tau_a = 1 + 0.2274w^2 - 0.901w^4 10^{-2} + 0.491w^6 10^{-3} - 0.248w^8 10^{-4} \ldots (6)$$

$$R_a/v_a = 1 + 0.1137w^2 - 0.338w^4 10^{-2} + 0.155w^6 10^{-3} - 0.068w^8 10^{-4} \ldots (7)$$

If we assume the components of the cosmic velocities to be distributed according to a Laplacean error curve of the first type we must write:

$$f(\tau)d\tau = \frac{1}{2k}\exp(-|\tau|)d\tau,\ \tau_a = 1/k \quad \ldots\ldots\ldots\ldots (8)$$

Integration in this case gives immediately

$$v_\lambda = V\sin\lambda + \frac{1}{k}\exp(-kV\sin\lambda)$$

$$R_\lambda = V\cos\lambda + \frac{1}{k}\exp(-kV\cos\lambda)$$

Subsequently integrating over all values of λ we get:

$$v_a = \frac{\pi}{4}V + \frac{1}{k}\int_0^{\pi/2} e^{-kV\sin\lambda}\sin\lambda d\lambda$$

$$R_a = \frac{1}{2}V + \frac{1}{k}\cdot\frac{1}{kV}(1-e^{-kV})$$

Again expanding into power series, and employing as independent variable w, the ratio between V and $\tau_a = 1/k$, $w = kV$, we obtain

$$\frac{v_a}{\tau_a} = 1 + \frac{2}{3}\cdot\frac{w^2}{2!} + \frac{2.4}{3.5}\frac{w^4}{4!} + \frac{2.4.6.w^6}{3.5.7.6!} + \ldots \quad -\frac{\pi}{2}\left[\frac{1.3}{2.4}\frac{w^3}{3!} + \frac{1.3.5}{2.4.6}\frac{w^5}{5!} + \ldots\right.$$

$$\frac{R_a}{V_a} = 1 + \frac{w^2}{3!} - \frac{w^3}{4!} + \frac{w^4}{5!} - \frac{w^5}{6!} + \ldots\ldots$$

and numerically:

$$v_a/\tau_a = 1 + 0.3333w^2 - 0.982w^3 10^{-1} + 0.222w^4 10^{-1} - 0.409w^5 10^{-2} + 0.630w^6 10^{-3} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$R_a/V_a = 1 + 0.1667w^2 - 0.417w^3 10^{-1} + 0.083w^4 10^{-1} - 0.139w^5 10^{-2} + 0.200w^6 10^{-3} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The series (6), (7), (9) and (10) are all absolutely convergent for all values of the argument w. The numerical sums of these series are given in table 1 for different values of w, covering almost the entire range needed in practice.

We may here recall that we derived[3] the following expression relating the observed mean tangential velocity T_a and the mean T_0 corrected for drift motion.

$$T_a/T_0 = 1 + \frac{a}{3} - \frac{a^2}{30} + \frac{a^3}{210} - \frac{a^4}{1512}\ldots\ldots$$

where a now means $a = h^2V^2 = w^2/\pi$.

Comparing this with equation (5), we see

$$T_a/T_0 = R_a/v_a$$

and also

$$T_a/R_a = T_0/v_a = \pi/2.$$

This result, which is obviously true, regardless of the type of frequency function assumed, may serve as a numerical check on the different series derived for T_a and R_a.

TABLE 1

	MAXWELL		LAPLACE	
w	v/τ	R/v	v/τ	R/v
0.0	1.000	1.000	1.000	1.000
0.2	1.012	1.006	1.012	1.006
0.4	1.036	1.018	1.045	1.023
0.6	1.081	1.041	1.100	1.052
0.8	1.141	1.071	1.174	1.090
1.0	1.219	1.110	1.254	1.132
1.25	1.335	1.171	1.39	1.20
1.50	1.469	1.240	1.50	1.26
2.00	1.792	1.409	1.81	1.43

[1] These PROCEEDINGS, **11**, 87, 1925.
[2] *Nature*, **114**, 610, 1924.
[3] These PROCEEDINGS, **11**, 90, 1925.

ASTROPHYSICAL DATA BEARING ON THE RELATIVE ABUNDANCE OF THE ELEMENTS

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1. The physical interpretation of stellar spectra has recently yielded interesting results under the combined attack of thermodynamics and spectrum theory. The extension, by Fowler and Milne,[1] of Saha's original application[2] of the laws of thermal ionization, has been subjected by Menzel[3] and by the writer[4] to an extensive observational test. The agreement with observation is in general satisfactory, and the deviations are such as to suggest the necessity of improving an essentially valid theory by a second approximation.[5]

2. The original discussion by Saha[2] made use of the "marginal appearance" of a line (the class at which the line is on the point of disappearing) in assigning a stellar temperature scale on ionization principles. Fowler and Milne[6] pointed out that the temperature of disappearance is a function of the relative abundance of the element in question, and their form of

the theory eliminated the unknown abundance factor by treating the maximum, rather than the marginal appearance of a line, as the important quantity. They pointed out, however, that the application of their theory should lead to conclusions as to relative abundance. A preliminary attempt to apply estimates of marginal appearance, made with a homogeneous collection of stellar spectra at the Harvard Observatory, is contained in the present paper. The computations have been made by Mr. R. O. Suter.

3. The outline of the method is as follows. The fraction of the total number of atoms of a given kind which is active in absorption at a given temperature is obtained, for subordinate series, from the formula[7]

$$n_r = \frac{q_r\, e^{-(\chi_1-\chi_r)/kT}}{b(T) + 0.332\sigma/P_e\,(T^{5/2}\, e^{-\chi_1/kT})}$$

n_r = fraction of active atoms (the "fractional concentration" of Fowler and Milne)
q_r = the weight factor = 1 in the present application
$\chi_1, \chi_1-\chi_r$ = ionization and resonance potentials, respectively
k = Boltzmann's constant = 1.37×10^{-16}
T = absolute temperature
$b(T)$ = the partition function = 1 in the present application
σ = number of valence electrons (spectroscopic)
P_e = partial electron pressure = 1.31×10^{-4} atm.[8]

For principal series of the ionized atom use is made of the similar expression

$$n_r = \frac{q_r\, e^{-(\chi_1-\chi_r)/kT}}{b(T) + \dfrac{P_e}{0.332\sigma}\, T^{-5/2}\, e^{\chi_1/kT} + \dfrac{0.332\sigma'}{P_e}\, T^{5/2}\, e^{-\chi_1'/kT}}$$

where primes denote that the corresponding quantities are referred to the ionized atom. It is assumed that at marginal appearance the same number of atoms is involved for all elements. Fowler and Milne have already suggested as a plausible first assumption that "other things being equal, the intensity of a given absorption line in the stellar spectrum is proportional to the concentration of atoms in the stellar atmosphere capable of absorbing the line." [6] The relative abundance of the corresponding atoms should then be given directly by the reciprocals of the fractional concentrations at marginal appearance.

4. The method suggested is to be regarded as a first approximation. It is probable that correcting factors exist. Certain assumptions involved in the application of the theory may well be enumerated here.

(*a*) Uniformity of composition of stellar atmospheres. The success

of the observational test of the theory of thermal ionization has established with considerable certainty that this assumption is of almost universal validity.[9]

(*b*) The relative absorbing efficiency of the atoms discussed is of the same order for all, under the conditions in the stellar reversing layer.

(*c*) The effects of collisions and of nuclear fields are negligible at the temperatures and pressures concerned.[10]

(*d*) At marginal appearance all lines are unsaturated.[11]

5. As a check on the method, the relation between the theoretical fractional concentration of the lines originating at different energy levels of the same atom, and the observed solar intensity of the strongest line originating from this level, has been examined for several elements. The results are given in table 1. The element, the excitation potential, the computed fractional concentration for $T = 5600°$, and the solar intensity from Rowland's tables, are given in successive columns. The theory appears to be in very satisfactory accord with observation.

TABLE 1

ELEMENT	EXCITATION POTENTIAL	COMPUTED LOG n_r	SOLAR INTENSITY	ELEMENT	EXCITATION POTENTIAL	COMPUTED LOG n_r	SOLAR INTENSITY
Chromium	0.0	$\bar{1}$.2038	10	Calcium	0.0	$\bar{2}$.6733	20
	0.94	$\bar{2}$.3769	5		1.88	$\bar{4}$.9743	15
	1.02	$\bar{2}$.3100	5		2.53	$\bar{4}$.4057	8
	2.89	$\bar{4}$.6622	2		2.70	$\bar{4}$.2644	5
Titanium[12]	0.0	$\bar{1}$.8894	5		2.92	$\bar{4}$.0712	4
	0.82	$\bar{2}$.1447	4	Iron[13]	0.0	$\bar{2}$.6605	40
	0.90	$\bar{2}$.0797	3		0.94	$\bar{3}$.8646	30
	1.05	$\bar{3}$.9501	3		1.54	$\bar{3}$.3137	30
	1.44	$\bar{3}$.8137	3		2.19	$\bar{4}$.7019	8
	1.50	$\bar{3}$.5426	2		2.46	$\bar{4}$.4458	10
	1.87	$\bar{3}$.2116	1		2.84	$\bar{4}$.0755	8
	1.98	$\bar{3}$.2098	1		2.96	$\bar{4}$.0222	7
	2.08	$\bar{3}$.0355	0		3.25	$\bar{5}$.7744	7
	2.16	$\bar{4}$.9501	1		3.38	$\bar{5}$.6605	6
	2.24	$\bar{4}$.8894	2		3.64	$\bar{5}$.3929	8
	2.26	$\bar{4}$.8532	0		4.13	$\bar{6}$.9393	..
	2.28	$\bar{4}$.8339	0		4.23	$\bar{6}$.8646	..
	2.33	$\bar{4}$.7979	0		4.35	$\bar{6}$.8366	..
	2.39	$\bar{4}$.7467	00		4.40	$\bar{6}$.7019	..
	2.47	$\bar{4}$.6747	..		..	....	..
	2.56	$\bar{4}$.5970	000		..	....	..
	2.67	$\bar{4}$.5229	000		..	....	..

6. The method used in obtaining the estimates of line-intensity which are employed in the present paper is that of *Harvard Circulars* 252 and 256, and calls for no special remark. The class at which a line is last seen is a function of the dispersion, definition and density of the plates used. In order to make the data as homogeneous as possible, all the spectra,

with the few exceptions mentioned, were measured with the same dispersion (48.6 mm. between Hϵ and Hβ) and only images of comparable density and definition were used.

7. The data on marginal appearance will be found in table 2. Successive columns contain the atomic number and atom, the series relations, the wave length of the line used, and the Draper classes of appearance, of maximum and of disappearance. Asterisks in the last column denote the ultimate lines of the neutral atom, which are strongest at low temperatures and have no maximum. Estimates made by Menzel[3] on a similar series of plates are indicated by †; those marked by ‡ are taken from specially stained plates made with slightly smaller dispersion, and § denotes an estimate suggested by Russell.

TABLE 2

ATOM	SERIES	LINE	CLASSES			ATOM	SERIES	LINE	CLASSES		
1 H	$1S$–$2P$	4340	–	A3	–	22 Ti	$1f$–f_z	3999	*	*	A2
2 He	1π–3δ	4471	B9	B3	O		$1f$–g	4862	*	*	A2†
	$1S$–$2P$	5015	B9	B3	O			4867	*	*	A2†
	$1P$–$4D$	4388	B9	B3	O			4856	*	*	A2†
He$^+$	$4F$–$9G$	4542	O	O	–		$1f^5$–f_z^5	4536	–	–	A5†
3 Li	1σ–1π	6707	*	–	–			4535	–	–	A5†
6 C$^+$	2δ–3ϕ	4267	B9	B3	O	23 V	$1g^4$–g_z^4	4333	*	*	F0
11 Na	1σ–1π	5889	*	*	A0‡			4330	*	*	F0
		5896	*	*	A0‡	24 Cr	$1s^7$–$1p^7$	4290	*	*	A2
12 Mg	$1p$–$2d$	5184	–	?	A0‡			4275	*	*	A2
		5173	–	?	A0‡			4254	*	*	A2
		5167	–	?	A0‡		$1s^6$–$1p^5$	4497	–	M1	A7
		3838	–	?	A0	25 Mn	$1s$–$1p$	4034	*	*	A2
		3832	–	?	A0			4033	*	*	A2
		3829	–	?	A0			4030	*	*	A2
Mg$^+$	2δ–3ϕ	4481	–	A3	B0		$1d^6$–d_z^6	4084	–	K2	A3
13 Al	1π–1σ	3962	*	*	A0			4041	–	K2	A3
		3944	*	*	A0	26 Fe	$1f^3$–g^3	4325	–	K2	A2
14 Si		3905	–	G0	A2		$1f^3$–f^3	4072	–	K0	A0
Si$^+$		4128	F0	A0	O	30 Zn	$1p$–$1s$	4811	G5	G0	A7†
		4131	F0	A0	O			4722	G5	G0	A7†
19 K	1σ–1δ	4642	*	*	F8§	38 Sr	$1S$–$1P$	4607	*	*	F0
		4641	*	*	F8§	Sr$^+$	1σ–1π	4078	–	K2	A0
20 Ca	$1S$–$1P$	4227	*	*	B9	Ba$^+$	1σ–1π	4555	–	?	A2†
	$1p$–$2d$	4455	–	K2	F0						
Ca$^+$	1σ–1π	3933	–	–	B0						

8. The relative abundance of the atoms which give rise to the lines tabulated may be deduced, by the method of Section 3, if the temperature scale is known. That here used (table 3) is the ionization temperature scale given by the writer.[4] The substantial agreement of this scale with those obtained by other methods has indicated the validity of the theory of thermal ionization from which it is derived.

TABLE 3

DRAPER CLASS	TEMPERATURE	DRAPER CLASS	TEMPERATURE	DRAPER CLASS	TEMPERATURE	DRAPER CLASS	TEMPERATURE
Ma	3000°	G0	5600°	A2	10000°	B3	16000°
K5	3500	F8	6250	A0	11000	B2	17500
K2	3750	F5	7000	B9	12500	B1	18500
K0	4000	F0	7500	B8	13500	B0	20000
G5	5000	A5	9000	B5	15000	O	up to 35000

9. Table 4 contains, in successive columns, the atomic number, atom and series relations, the logarithm of the fractional concentration n_r at marginal appearance, and the logarithm of the relative abundance a_r, deduced by the method of Section 3.

TABLE 4

AT. NO.	ATOM	SERIES	LOG n_r	LOG a_r	AT. NO.	ATOM	SERIES	LOG n_r	LOG a_r
1	H	$1S$–$2P$	$<\overline{11}$	>11		Ca^+	1σ–1π	$\overline{5}.0$	5.0
2	He	1π–3δ	$\overline{9}.7$	8.3	22	Ti	$1f$–f_x	$\overline{5}.9$	4.1
		$1S$–$2P$	$\overline{9}.7$	8.3			$1f$–g	$\overline{5}.9$	4.1
		$1P$–$4D$	$\overline{9}.7$	8.3			$1f^5$–f_x^5	$\overline{5}.9$	4.1
	He^+	$4F$–$9G$	$<\overline{12}$	>12	23	V	$1g^4$–g_x^4	$\overline{3}.0$	3.0
3	Li	1σ–1π	–	–			$1d^6$–f^6	$\overline{4}.9$	3.1
6	C^+	2δ–3ϕ	$\overline{5}.5$	4.5	24	Cr	$1s^7$–$1p^7$	$\overline{4}.0$	4.0
			$\overline{6}.5$	5.5			$1s^5$–$1p^5$	$\overline{4}.2$	3.8
11	Na	1σ–1π	$\overline{6}.8$	5.2	25	Mn	$1s^6$–$1p^6$	$\overline{5}.6$	4.4
12	Mg	$1p$–$2d$	$\overline{6}.4$	5.6			$1d^6$–d_x^4	$\overline{5}.2$	4.8
	Mg^+	2δ–3ϕ	$\overline{6}.5$	5.5	26	Fe	$1f^3$–g^3	$\overline{5}.2$	4.8
13	Al	1π–1σ	$\overline{5}.0$	5.0			$1f^3$–f^3	$\overline{5}.1$	4.9
14	Si		$\overline{5}.2$	4.8	30	Zn	$1p$–$1s$	$\overline{5}.7$	4.3
	Si^+		$\overline{5}.1$	4.9				$\overline{5}.9$	4.1
	Si^{+++}		$\overline{6}.0$	6.0	38	Sr	$1S$–$1P$	$\overline{2}.2$	1.8
19	K	1σ–1δ	$\overline{4}.5$	3.5		Sr^+	1σ–1π	$\overline{2}.5$	1.5
20	Ca	$1S$–$1P$	$\overline{5}.1$	4.9	54	Ba^+	1σ–1π	$\overline{2}.9$	1.1
		$1p$–$2d$	$\overline{5}.2$	4.8					

10. The relative abundance of atomic species has long been of recognized significance, and several previous astrophysical estimates have been made. Russell[14] published a discussion based on the total intensity of absorption by different elements in the Sun, and H. H. Plaskett[15] later made estimates of relative abundance based on the theory of thermal ionization. The order of abundance derived by Russell's admittedly qualitative method for the solar atmosphere showed a strong and presumably significant parallelism with the order found for the Earth's crust. This parallelism is perhaps somewhat surprising, since neither source is necessarily representative of the composition of the body of which it is a part. The estimates of the present paper confirm this similarity, as will be seen from table 5. Successive columns contain the atomic number, the atom, the relative stellar abundance and the terrestrial abundance, both expressed

in atoms. Column 4 is deduced from the tables of Clarke and Washington[16] by dividing the tabular values by the corresponding atomic weights. The numbers expressing the stellar abundance are percentages, calculated on the assumption that the stellar and terrestrial elements discussed form the same fraction of the total material present. This reduces the two last columns to a form in which they are directly comparable, but no great importance is attached to the absolute percentages in the third column. It will be seen that only in the case of zinc are the numbers of different orders of magnitude.

TABLE 5

AT. NO.	ATOM	RELATIVE STELLAR ABUNDANCE	TERRESTRIAL ABUNDANCE (EARTH'S CRUST)
14	Si	5.7	16.2
11	Na	5.7	2.02
12	Mg	4.2	0.42
13	Al	3.6	4.95
6	C	3.6	0.21
20	Ca	2.9	1.50
26	Fe	2.5	1.48
30	Zn	0.57	0.0011
22	Ti	0.43	0.241
25	Mn	0.36	0.035
24	Cr	0.29	0.021
19	K	0.11	1.088
23	V	0.05	0.0133
38	Sr	0.002	0.0065
54	Ba	0.0005	0.0098
3	Li	0.0000	0.0829

Hydrogen and helium are omitted from the table. The stellar abundance deduced for these elements is improbably high, and is almost certainly not real. Russell and Compton[17] have suggested that the anomalous astrophysical behavior of the Balmer lines may be attributed to metastability, an interpretation which would also explain the great apparent abundance of the element in stellar atmospheres. The abundance of helium has been shown by H. H. Plaskett,[15] in his investigation of class O stars, to be surprisingly great. The cause of the discrepancy is unexplained. Aston[18] and Jeffreys[19] have commented on the terrestrial deficiency of the rare gases.

11. The third and fourth columns of table 5 show a parallelism which appears to have some physical significance. Recent work[20] has extended our knowledge of the constitution of the Earth, so that its total percentage composition may be inferred. These estimates raise iron, nickel and magnesium on the terrestrial scale, so that the order of abundance, as given by Washington, is Fe, O, Si, Mg, Ni, Ca, Al. A quantitative estimate is not made. Owing to the high atomic weight of iron, its percentage by atoms is not altered to the same extent as is the percentage by weight.

12. It is outside the scope of the present paper to do more than refer to the relation of these estimates to the work of Aston[21] and of Harkins[22] on the abundance of atomic species, with its bearing on questions of nuclear stability. The result is also clearly relevant to theories of the origin of the Earth.

[1] *M. N. R. A. S.*, **81**, 403 (1923); *Ibid.*, **82**, 499 (1924).
[2] *Proc. Roy. Soc.*, **99A**, 135 (1921).
[3] *Harv. Obs. Circ.* 258 (1924).
[4] *Harv. Obs. Circ.* 252 and 256 (1924).
[5] Stewart, *American Astronomical Society*, January, 1925.
[6] *Loc. cit.* **81**, 403 (1923).
[7] *Loc. cit.* **81**, 408 (1923).
[8] Cf. Russell and Stewart, *Ap. J.*, **59**, 197 (1924).
[9] Milne, *Proc. Phys. Soc. Lond.*, **36**, 94 (1924).
[10] Fowler, *Phil. Mag.*, **47**, 257 (1924).
[11] Russell, *Pop. Ast.*, **32**, 620 (1924).
[12] The ionization potential of iron is assumed to be 7.5, possibly too low an estimate. Its value affects the absolute but not the relative fractional concentrations.
[13] Russell, *Ap. J.*, in press.
[14] *Science*, **39**, 791 (1914).
[15] *Pub. Dom. Ast. Obs.*, **1**, 325 (1922).
[16] *Proc. Nat. Acad. Sci.*, **8**, 108 (1922).
[17] *Nature*, **114**, 86 (1924).
[18] *Ibid.*, **114**, 786 (1924).
[19] *Ibid.*, **114**, 934 (1924).
[20] Washington, *Jour. Wash. Acad. Sci.*, **14**, 333, 435 (1924).
[21] *Nature*, **113**, 393 (1924).
[22] *Jour. Frank. Inst.*, **194**, 165 (1920).